Australian Curriculum Linked Lessons

Derek Hurrell (Notre Dame University, Fremantle, WA)

In providing a continued focus on tasks and activities that help to illustrate key ideas embedded in the new Australian Curriculum, in this issue we focus, on Algebra in the Number and Algebra strand.

	Number & Algebra	Measurement & Geometry	Statistics & Probability
Understanding			
Fluency			
Reasoning			
Problem Solving			

This is a small unit of work on a very important mathematical understanding, that of equivalence. Equivalence is mostly associated with the strand of Number and Algebra and is fundamental in both of the substrands. Equivalence is considered a key and foundational concept of algebra, and whereas an assumption might be made that equivalence is a relatively 'easy' concept; studies have shown that when asked to balance an equation such as "8 + 4 = ___ + 5," students have substantial difficulties. The activities provided can be modified to meet the requirements of particular year level descriptors and although the descriptors articulated here from the Australian Curriculum start at Year 4, the work needs to, and can be developed, from a much younger age.

Number and Algebra — Patterns and algebra

Year 4

Patterns and algebra

Use equivalent number sentences involving addition and subtraction to find unknown quantities (ACMNA083)

- writing number sentences to represent and answer questions such as: 'When a number is added to 23 the answer is the same as 57 minus 19. What is the number?'
- using partitioning to find unknown quantities in number sentences

Year 5

Patterns and algebra

Use equivalent number sentences involving multiplication and division to find unknown quantities (ACMNA121)

- using relevant problems to develop number sentences

Year 7

Patterns and algebra

Extend and apply the laws and properties of arithmetic to algebraic terms and expressions (ACMNA177)

- identifying order of operations in contextualised problems, preserving the order by inserting brackets in numerical expressions, then recognising how order is preserved by convention
- moving fluently between algebraic and word representations as descriptions of the same situation

Linear and non-linear relationships

Solve simple linear equations (ACMNA179)

- solving equations using concrete materials, such as the balance model, and explain the need to do the same thing to each side of the equation

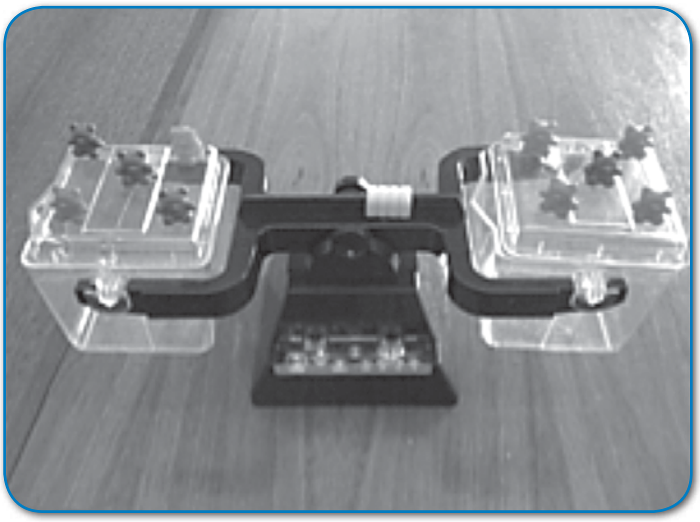
An understanding of mathematical equivalence requires the student to understand that the values on either side of an equals sign are the same. This understanding is different and distinct from knowledge of numerical equivalence.

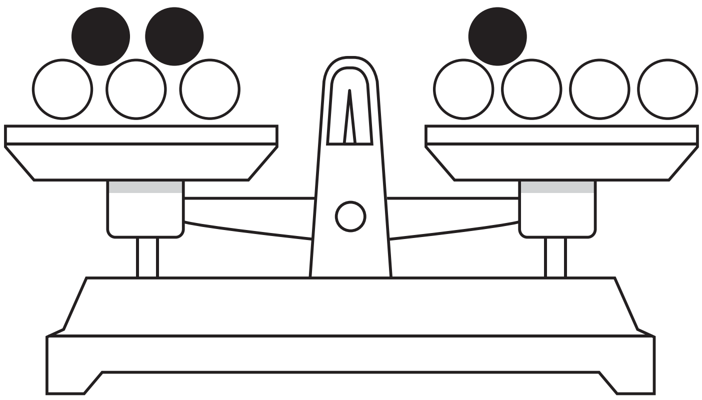
In order to develop a strong conceptual understanding, a good place to start with equivalence is to employ concrete materials—and a set of balance scales is a powerful tool. The scales need to be checked by the students to make sure they display equivalence by the trays or pans being "equal" height from the work top. If the balance scales are slightly "out" (which can happen if cheaper sets are purchased) a small amount of "blutac" (or something similar) attached to the underside of one of the pans can usually correct the difference. Any uniform materials can then be used to develop the concept. A favourite for many students are the plastic tiny teddies.

Start by using only two colours. "Set out 3 blue teddies and 2 red teddies on one side of the balance scale. How many different ways can you balance the teddies by placing teddies on the other side of the scale?"

The optimal outcome would be that the students systematically work out all of the possibilities, that is: 5 blue teddies and 0 red teddies, 4 blue teddies and 1 red teddy, 3 etc. Of course some students may not be so systematic and may produce all of the possibilities through a trial and correction basis. The different colours are not essential, but help the students to visualise the two addends and help strengthen the property of commutativity of addition. It is also very important at this stage to articulate the notion that the equals sign separates two equivalent expressions and does not represent "What I need to do" on the left hand side of the equals sign and "an answer" on the right hand side. Reversing the process so that the initial statement is in the left hand pan and the 'building' occurs in the right hand pan further strengthens the required understandings.

Once the concept that the equal sign shows equivalence or balance is established by the use of the physical balance scales, then pictorial representations can be introduced (should you wish to use them the final page of this article contains some pictures of scales, etc.).





It is a clarifying step for the students to record their results as statements (i.e., 3 blue + 2 red = 1 blue + 4 red) and then tell a ‘story,’ that is, verbally articulate what all of the “bits” are before they write the equation as 3 + 2 = 1 + 4. Later again, the representation can be made more abstract by creating the balances in a more stylised manner; for example:

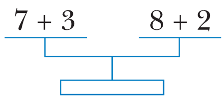
3 + 2 = 1 + 4 ________ = ________ ________ = ________ ________ = ________

Draw and write three more equivalent statements.

2 + 2 = 1 + 3 ________ = ________ ________ = ________ ________ = ________

Draw and write three more equivalent statements.

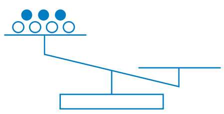
A shift can then be made to using numbers instead of representations, that is moving from:



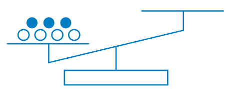
to 8 + 4 = 9 + 3 and then 6 + 2 = 4 + ____ and 4 + ___ = 6 + 1 and

then 7 + 6 = ________ and _________ = 4 + 9.

This can be a very good opportunity to keep the analogy of the balance scales operating to illustrate inequality. Teachers can follow the same procedure used in developing equality to illustrate inequality. For instance the representations of balance scales could be as such:



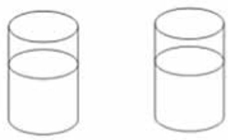
and



Other pictorial representations should be introduced so that the students do not labour under the misapprehension that a balance scale is the only way to show equivalence.

___+___=___+___

___+___=___+___

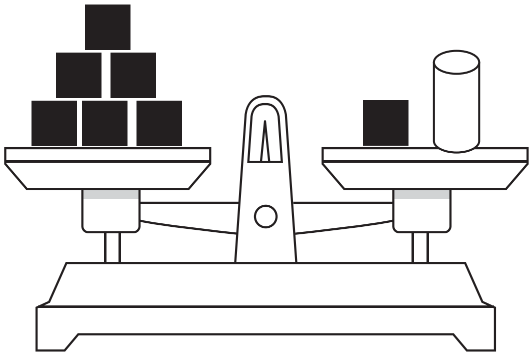


___+___=___+___

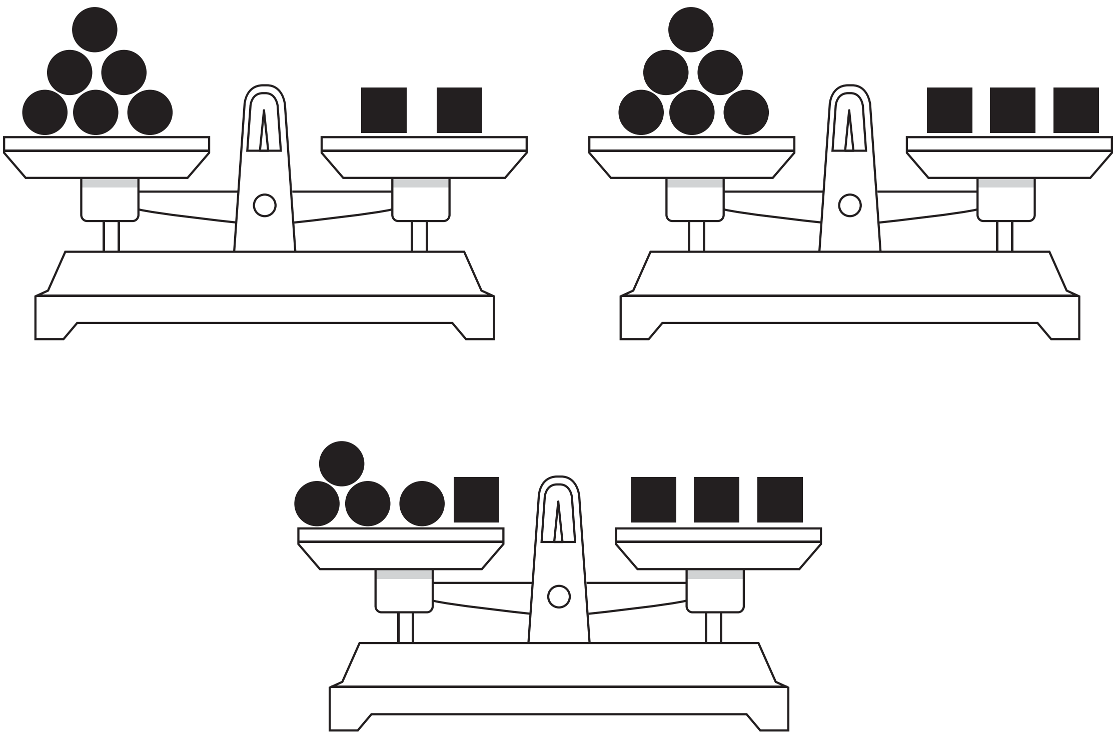
Create an equation where the total number of shells and starfish are the same in each tank.

Previously, the concept had been developed using only the operation of addition but the understanding needs to be developed that the same principles for equality are required for all of the operations and indeed where the operations are mixed. For example: $3 + 4 = 12 - 5$ or $3 \times 7 = 42 \div 2$ and eventually equations such as $6^2 + 9 - 4 = 10 \times 5 - 8 - 1$.

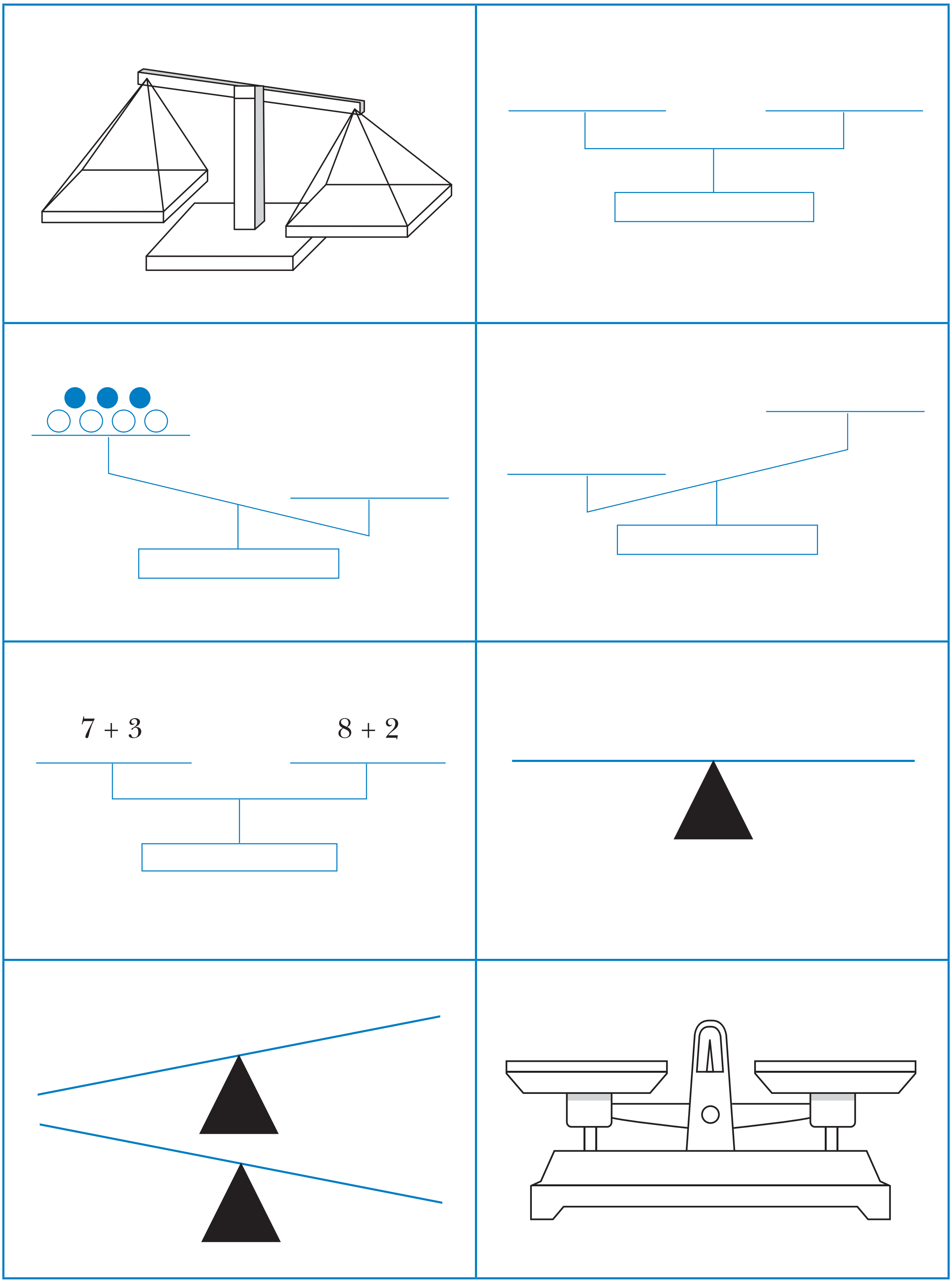
A further development of this concept is where the two expressions may not have the same appearance and where the students are required to determine the value of an object; for example: if a cube is worth one, what is a cylinder worth?



If a sphere has a value of one, what is the value of a cube?



Two sites that can be utilised to provide some supporting ICT are the Learning Federation's website http://econtent.thelearningfederation.edu.au/ec/p/home which supports such activities as "Balance the Cup" in the algebra section and the Illuminations website http://illuminations.nctm.org/Activities.aspx?grade=2 which provides activities such as "Pan Balance" for shapes, numbers and expressions in the "Activities" section Grades 6–8.



We would like to encourage any teachers trying these ideas with their classes to send in a short paragraph explaining what happened. Samples of children's work illustrating how they tackled these tasks would be appreciated. Likewise any assessment schemes that could be shared among colleagues would be welcomed.